$$\left(\begin{array}{ccc|c} 1 & 1/2 & 1/2 & 1/2 \\ 0 & -1/2 & -3/2 & 1/2 \end{array}\right) \xrightarrow{R_2 \mapsto -2R_2} \left(\begin{array}{ccc|c} 1 & 1/2 & 1/2 & 1/2 \\ 0 & 1 & 3 & -1 \end{array}\right)$$

$R_1 \mapsto R_1 - \frac{1}{2}R_2$

$$\left(\begin{array}{ccc|c} 1 & 0 & -1 & -1 \\ 0 & 1 & 3 & -1 \end{array}\right) \checkmark$$

0 means pivot point is unchanged!

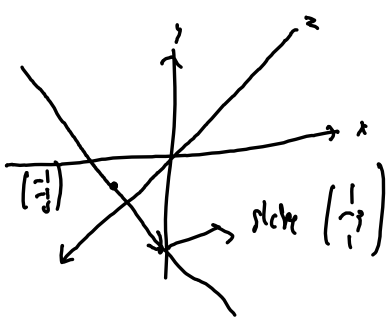
$$x - z = -1$$
$$y + 3z = -1$$

$$x = z - 1$$
$$y = -3z - 1$$

solution $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} z-1 \\ -3z-1 \\ z \end{pmatrix}$

How many sols? ∞ !

$$\begin{pmatrix} z-1 \\ -3z-1 \\ z \end{pmatrix} = \begin{pmatrix} z \\ -3z \\ z \end{pmatrix} + \begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix}$$
$$= \begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix} + z\begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix}$$



$$2x + y + z = 1$$
$$3x + y = 2$$

$$\longleftrightarrow \quad \begin{array}{c} \left(\begin{array}{ccc|c} 2 & 1 & 1 & 1 \\ 3 & 1 & 0 & 2 \end{array}\right) \\ \begin{array}{ccc} x & y & z \end{array} \end{array}$$

$$R_1 \longmapsto \tfrac{1}{2} R_1$$

$$\left(\begin{array}{ccc|c} 1 & 1/2 & 1/2 & 1/2 \\ 3 & 1 & 0 & 2 \end{array}\right) \xrightarrow{R_2 \longmapsto R_2 - 3R_1} \left(\begin{array}{ccc|c} 1 & 1/2 & 1/2 & 1/2 \\ 0 & -1/2 & -3/2 & 1/2 \end{array}\right)$$

$1/2$

$2 - 3 \cdot \frac{1}{2}$

$$R_2 \longmapsto -2 R_2$$

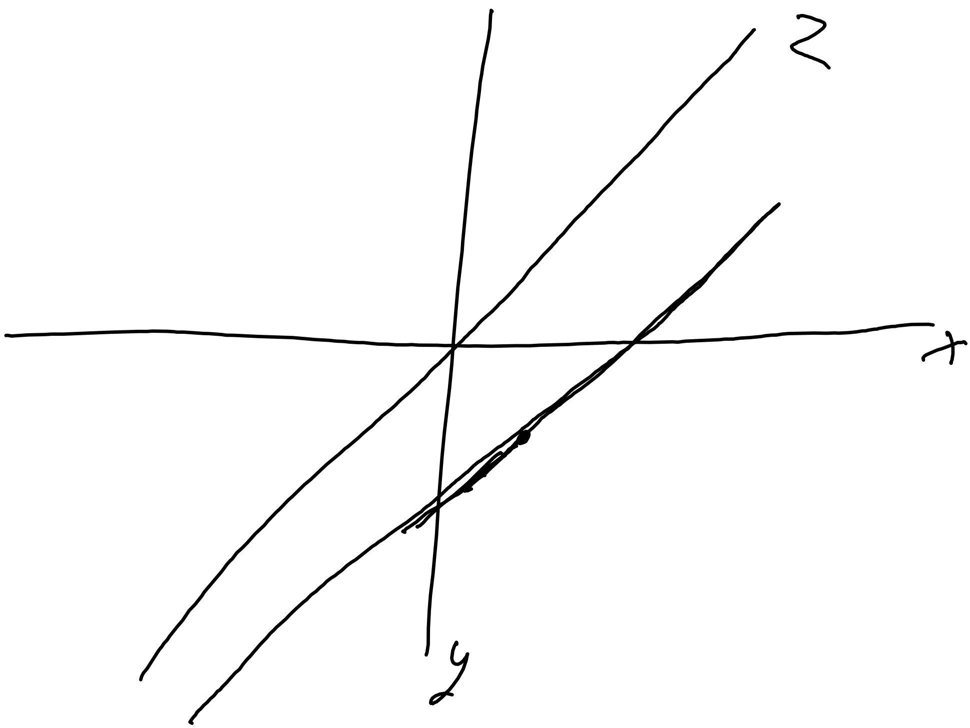
$$\left(\begin{array}{ccc|c} 1 & 1/2 & 1/2 & 1/2 \\ 0 & 1 & 3 & -1 \end{array}\right) \xrightarrow{R_1 \longmapsto R_1 - \frac{1}{2} R_2} \left(\begin{array}{ccc|c} 1 & 0 & -1 & 1 \\ 0 & 1 & 3 & -1 \end{array}\right)$$

$$2x + y + z = 1$$
$$3x + y = 2$$
$$\downarrow$$
$$x + \tfrac{1}{2} y + \tfrac{1}{2} z = 1$$
$$3x + y = 2$$

$$x - z = 1$$
$$y + 3z = -1.$$

$$x = z + 1$$
$$y = -3z - 1, \quad \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} z+1 \\ -3z-1 \\ z \end{pmatrix}$$

$$\begin{pmatrix} z+1 \\ -3z-1 \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} + z \begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix}$$



$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & a \\ 0 & 1 & 0 \end{pmatrix} \xrightarrow{R_3 \leftrightarrow R_2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & d \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 0 & d \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

d mi d mi

V_1 , V_2

boat river

speed · time = distance

1 length

$$(V_1+V_2)\,40 = 1$$
$$(V_1-V_2)\,60 = 1$$

$$\frac{40}{60} = \frac{2}{3}$$

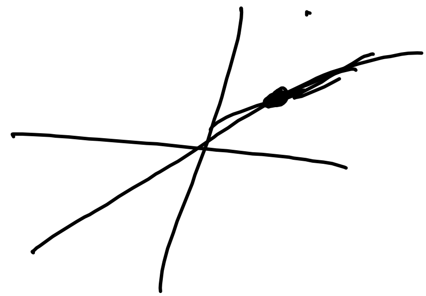
$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 1 & * \\ 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

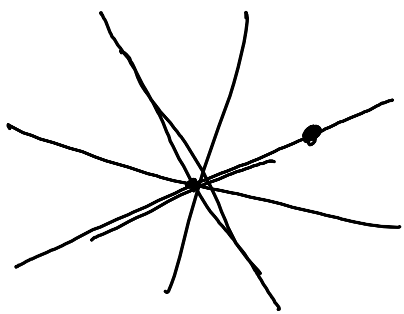
$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 1 & * \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

?⇑ $x = y$ ✓

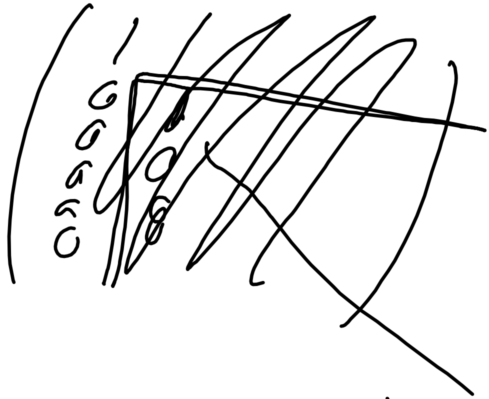
$\Downarrow$

$x^2 = y^2$ ✓



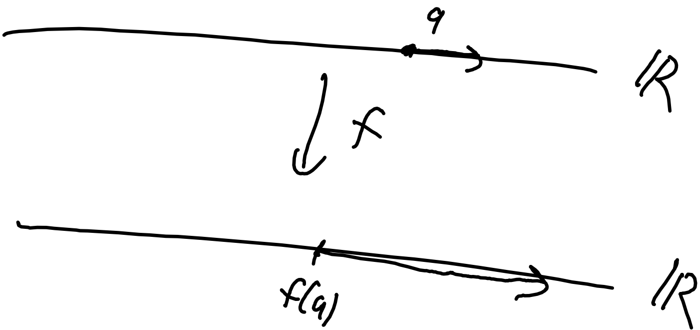


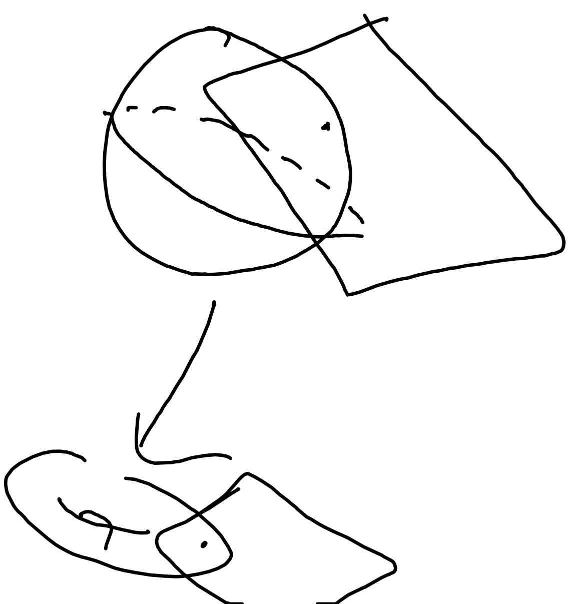
1.2.18



$f'(a) =$

$f(\vec{x}) \approx f(\vec{a}) + \underbrace{f'(a)}(\vec{x} - \vec{a})$





$$\begin{cases} 2x+y=5 \\ x-y=7 \end{cases}$$

$$\underset{\substack{\| \\ A}}{\begin{pmatrix} 2 & 1 \\ 1 & -1 \end{pmatrix}} \underset{\substack{\| \\ \vec{x}}}{\begin{pmatrix} x \\ y \end{pmatrix}} = \underset{\substack{\| \\ \vec{b}}}{\begin{pmatrix} 5 \\ 7 \end{pmatrix}}$$

$$\underline{A\vec{x}=\vec{b}}$$

1.2.50

$x_2 = \frac{1}{2}(x_1 + x_3) \leftarrow$ $\qquad \frac{1}{2}x_1 + \frac{1}{2}x_3 - x_2 = 0$

$x_3 = \frac{1}{2}(x_2 + x_4)$

$x_{n-1} = \frac{1}{2}(x_{n-2} + x_n)$

$$\begin{array}{c}
\begin{bmatrix}
1/2 & -1 & 1/2 & 0 & \cdots & 0 & 0 \\
0 & 1/2 & -1 & 1/2 & 0 & \cdots & 0 \\
0 & 0 & 1/2 & -1 & 1/2 & 0 \cdots & 0 \\
\vdots & & \ddots & \ddots & \ddots & & \vdots \\
0 & 0 & 0 & \cdots & 1/2 & -1 & 1/2
\end{bmatrix} \\
x_1 \quad x_2 \quad x_3 \quad \cdots \quad x_{n-2} \quad x_{n-1} \quad x_n \\
\underbrace{\qquad\qquad\qquad\qquad\qquad\qquad}_{n}
\end{array}
\begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$n-2 \left[\begin{bmatrix}
1 & -2 & 1 & & & \\
 & 1 & -2 & 1 & & \\
 & & 1 & -2 & 1 & \\
 & & & \ddots & \ddots & \ddots \\
 & & & 1 & -2 & 1
\end{bmatrix} \right. \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}$$

$x_2 = \cdots$

$x_3 = \cdots$

$x_{n-1} = \cdots$

$$\boxed{x_n = 3x_{n+2} - 2x_{n+3}}$$

$$x_{n-1} = \frac{1}{2}(x_{n-2} + x_n)$$

2
3
5
7
9

$n=6$

$n = 4$

$x_n = 2x_{n-1} - x_{n-2}$

$$\begin{pmatrix} x_n \\ x_{n-1} \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x_{n-1} \\ x_{n-2} \end{pmatrix}$$

$(x-2)x+1$

$x^2 - 2x + 1$

$(x-1)^2$

$\sim \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

$$\left(\begin{array}{cccc|c} 1/2 & -1 & 1/2 & 0 & 0 \\ 0 & 1/2 & -1 & 1/2 & 0 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 1 & -2 & 1 & 0 & 0 \\ 0 & 1/2 & -1 & 1/2 & 0 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 1 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 \end{array}\right)$$

$R_1 \longmapsto R_1 + 2R_2$

$$\left(\begin{array}{cccc|c} 1 & 0 & -3 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 \end{array}\right) \longrightarrow$$

$x_1 = 3x_3$

$x_2 - 2x_3 + x_4 = 0$

$x_1 = 3x_3$

$x_2 = 2x_3 - x_4$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 3x_3 \\ 2x_3 - x_4 \\ x_3 \\ x_4 \end{pmatrix}$$

$$\left\{ x_3 \begin{pmatrix} 3 \\ 2 \\ 1 \\ 0 \end{pmatrix} + x_4 \begin{pmatrix} 0 \\ -1 \\ 0 \\ 1 \end{pmatrix} \;\middle|\; x_3, x_4 \in \mathbb{R} \right\}$$

$$
\begin{array}{cc}
n-2 & \left(\begin{array}{cccccc|c}
1 & -2 & 1 & & & & 0 \\
 & 1 & -2 & 1 & & & \vdots \\
 & & \ddots & \ddots & \ddots & & \vdots \\
 & & & 1 & -2 & 1 & 0
\end{array}\right) \\
 & n
\end{array}
$$

$$x_k = \frac{x_{k-1} + x_{k+1}}{2} \rightsquigarrow 2x_k = x_{k-1} + x_{k+1}$$

$$x_k = x_{k-1} - x_{k+1} - x_k$$

$$x_k - x_{k-1} = x_{k+1} - x_k$$

a: x_1 $\overset{d}{—}$ x_2 — x_3 — x_4 — x_5 —

$$\left\{ \left(\underset{x_1}{\underset{\parallel}{a}},\ \underset{x_2}{\underset{\parallel}{a+d}},\ \underset{x_3}{\underset{\parallel}{a+2d}},\ \ldots,\ \underset{x_n}{\underset{\parallel}{a+(n-1)d}} \right) \,\middle|\, a, d \in \mathbb{R} \right\}$$

$$
\begin{pmatrix}
1 & -2 & 1 & & & & \\
 & 1 & -2 & 1 & & & \\
 & & 1 & -2 & 1 & & \\
 & & & 1 & -2 & 1 & \\
 & & & \ddots & \ddots & \ddots & \\
 & & & & 1 & -2 & 1
\end{pmatrix}
$$

$$\downarrow$$

$$
\begin{pmatrix}
1 & 0 & -3 & 2 & & & \\
 & 1 & -2 & 1 & & & \\
 & & 1 & -2 & 1 & & \\
 & & & \ddots & \ddots & \ddots & \\
 & & & & 1 & -2 & 1
\end{pmatrix}
$$

$$
\begin{pmatrix}
1 & 0 & -3 & 2 & & & & \\
 & 1 & 0 & -3 & 2 & & & \\
 & & 1 & -2 & 1 & & & \\
 & & & 1 & -2 & 1 & & \\
 & & & & \ddots & \ddots & \ddots &
\end{pmatrix}
$$

$$\begin{pmatrix}
1 & 0 & -3 & 2 & & & & & \\
& 1 & 0 & -3 & 2 & & & & \\
& & 1 & 0 & -3 & 2 & & & \\
& & & \ddots & \ddots & \ddots & \ddots & & \\
& & & & & 1 & 0 & -3 & 2 \\
& & & & & & 1 & -2 & 1
\end{pmatrix}$$

2x3

$$\begin{pmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \not\leftrightarrow \begin{pmatrix} 1 & 0 & 4 \\ 0 & 1 & 3 \end{pmatrix}$$

$$\updownarrow \qquad\qquad\qquad \updownarrow$$

$$\begin{pmatrix} 1 & 3 & 0 \\ 0 & 0 & 1 \end{pmatrix} \not\leftrightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 5 \end{pmatrix}$$

$$\updownarrow$$

$$\begin{pmatrix} 1 & * & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\left\{ \begin{pmatrix} 1 & a & 0 \\ 0 & 0 & 1 \end{pmatrix} \,\middle|\, a \in \mathbb{R} \right\}$$

$$\left\{ \begin{pmatrix} 1 & 0 & a \\ 0 & 1 & b \end{pmatrix} \,\middle|\, \begin{matrix} a \in \mathbb{R} \\ b \in \mathbb{R} \end{matrix} \right\}$$

$$A \longrightarrow B$$

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \xrightarrow{R_1 \mapsto 2R_1} \begin{pmatrix} 2 & 4 \\ 3 & 4 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 2 & 4 \\ 3 & 4 \end{pmatrix}$$

$$R_i \longmapsto \lambda R_i$$

$$\begin{array}{c c} & \begin{array}{c} \\ i \end{array} \\ i & \underbrace{\begin{pmatrix} 1 & & & \\ & \ddots & & \\ & & \lambda & \\ & & & \ddots \\ & & & & 1 \end{pmatrix}}_{\operatorname{diag}(1, \dots, \lambda, \dots, 1)} A \end{array}$$

$$\begin{array}{c} i \end{array}$$

$$\begin{pmatrix} 1 & & & \\ & \ddots & & \\ & & 1/\lambda & \\ & & & \ddots \\ & & & & 1 \end{pmatrix} \qquad R_i \longmapsto \frac{1}{\lambda} R_i$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 4 \\ 1 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$$

$$\begin{pmatrix} 1 & \lambda \\ 0 & 1 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} \quad R_1 + \lambda R_2$$

$$^{-1} \quad \begin{pmatrix} 1 & -\lambda \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} a + c\lambda & b + d\lambda \\ c & d \end{pmatrix}$$

$$A \longrightarrow \mathrm{rref}(A)$$

$$A \downarrow E_1 A \downarrow E_2 E_1 A \downarrow \cdots \downarrow E_n \cdots E_1 A = \mathrm{rref}(A)$$

$$(*) \quad \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 9 \\ 6 \end{pmatrix}$$

$$\left(\begin{array}{cc|c} 1 & 2 & 9 \\ 3 & 4 & 6 \end{array}\right)$$

$$\underline{EAx = Ey}$$

$$\left(\begin{array}{cc|c} 3 & 4 & 6 \\ 1 & 2 & 9 \end{array}\right)$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 9 \\ 6 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 4 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 6 \\ 9 \end{pmatrix}$$